Graph Theory, Homework 1

Due Thursday, February 6

Recall that the binomial coefficient $\binom{n}{m}$ for m < n nonnegative integers, represents the number of distinct ways to choose m elements from a set with n elements.

By "graph" below, we mean "simple graph" unless specified otherwise!

- 1. Explain why the following statement is true: In any group of two or more people, there are always at least two people within the group who have the same number of friends within the group.
- 2. Explain why for any graph G with n vertices, the number of edges is no more than $\binom{n}{2}$.
- 3. Explain why, if a graph has n vertices and more than $\binom{n-1}{2}$ edges, it must be connected.
- 4. Is it possible to have a graph with 6 vertices whose degrees are 6, 3, 3, 2, 2, 1? Why or why not? What about a graph with 4 vertices whose degrees are 3, 3, 1, 1?