U, V chi square variables,

U w/ ν_1 degrees of freedom

V w/ ν_2 degrees of freedom

then $\rightarrow$ $\dfrac{(U/\nu_1)}{(V/\nu_2)}$ is a random variable w/ F-dist. w/ ν_1 & ν_2 degrees of freedom

<u>Main application:</u> if S_1^2 S_2^2 are sample variances from normal populations, independent samples, pop variances σ_1^2 σ_2^2 then

chi square w/ n-1 degrees of freedom

chi square w/ n_2-1 degrees

(typos! sorry)

$$\left.\frac{\left[\cancel{(n_1-1)}S_1^2/\sigma_1^2\right]/\cancel{(n_1-1)}}{\left[\cancel{(n_2-1)}S_2^2/\sigma_2^2\right]/\cancel{(n_2-\overset{1}{\cancel{2}})}}\right\} \text{F distribution w/ } n_1-1,\ n_2-\overset{1}{\cancel{2}} \text{ degrees of freedom}$$

$$= \left.\frac{(S_1^2/\sigma_1^2)}{(S_2^2/\sigma_2^2)}\right\} \text{F dist. random var}$$

Single population

σ^2 unknown $\quad \sigma_1^2 = \sigma_2^2 = \sigma^2$

$$\left(S_1^2/\sigma_1^2\right) \Big/ \left(S_2^2/\sigma_2^2\right) = \frac{\left(S_1^2/\sigma\right)}{\left(S_2^2/\sigma\right)} = \frac{S_1^2}{S_2^2} = F$$

$n_1 = 21$

$n_2 = 10$

an F dist. variable.

w/ 20 & 9 degrees of freedom.

$$P\left(S_2 \geq \tfrac{1}{2} S_1\right) = P\left(S_2^2 \geq \tfrac{1}{4} S_1^2\right)$$

$$= P\left(4 \geq S_1^2/S_2^2\right) = P(F \leq 4) = 98\%$$

value of cdf of F at 4.

S_1 = sample std. dev for store 1 $\quad n_1 = 12$

S_2 - - - — — $\quad$ — $\quad$ 2 $\quad n_2 = 12$

$\sigma_1 = 12 \quad \sigma_2 = 30$

$$P\left(\frac{S_2}{S_1} \geq 2\right)$$

$$= P\left(\frac{S_1}{S_2} \leq \frac{1}{2}\right)$$

$$= P\left(\left(\frac{S_1}{S_2}\right)^2 \leq \frac{1}{4}\right)$$

$$F = \frac{\left(S_1^2/\sigma_1^2\right)}{\left(S_2^2/\sigma_2^2\right)} = \left(\frac{S_1^2}{S_2^2}\right)\left(\frac{30}{12}\right)^2$$

11, 11 degrees of freedom.

$$F = \left(\frac{S_1}{S_2}\right)^2\left(\frac{25}{4}\right)$$

$$= P\left(\left(\frac{25}{4}\right)\frac{S_1^2}{S_2^2} \le \frac{25}{4}\cdot\frac{1}{4}\right) = P\left(F \le \frac{25}{16}\right) = 76\%$$

chance of successful demonstration.

Suppose have two normally distributed populations take samples size 21, 36 we'd like to get 90% confidence interval for σ_1^2/σ_2^2

$s_1^2 = 9 \qquad s_2^2 = 20 \qquad \sigma_1^2/\sigma_2^2$

Useful notation: f_{α,ν_1,ν_2}

$$P(F \le f_{\alpha,\nu_1,\nu_2}) = 1-\alpha$$

$$P(F > f_{\alpha,\nu_1,\nu_2}) = \alpha$$

$$f = f_{0.05,20,35} \qquad f' = f_{0.05,35,20}$$

Goal: find random vars R_{sm}, R_{big} (depending on S_1^2, S_2^2)

s.t. $P(\sigma_1^2/\sigma_2^2 > R_{big}) = 5\%$

$$P(\sigma_1^2/\sigma_2^2 < R_{sm}) = 5\%$$

$$P(R_{sm} < \sigma_1^2/\sigma_2^2 < R_{big}) = 90\%$$

$$F = \frac{(S_1^2/\sigma_1^2)}{(S_2^2/\sigma_2^2)}$$ F var w/ 20, 35 degrees of freedom

$$F' = \frac{(S_2^2/\sigma_2^2)}{(S_1^2/\sigma_1^2)}$$ F var. w/ 35 & 20 deg......

$$P(F > f) = 5\%$$ $$P(F' > f') = 5\%$$

$$P\left(\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} > f\right) = 5\%$$ σ_1^2/σ_2^2

$$5\% = P\left(\frac{S_1^2}{S_2^2}\,\frac{\sigma_2^2}{\sigma_1^2} > f\right) = P\left(\frac{1}{f}\,\frac{S_1^2}{S_2^2} > \frac{\sigma_1^2}{\sigma_2^2}\right)$$

$$95\% = P\left(\frac{\sigma_1^2}{\sigma_2^2} \geq \underbrace{\frac{1}{f}\,\frac{S_1^2}{S_2^2}}_{R_{sm}}\right)$$

$$f = f_{0.05,20,35} = 1.88.$$

$$r_{sm} = \left(\frac{1}{1.88}\right)\cdot\frac{S_1^2}{S_2^2} = \frac{1}{1.88}\,\frac{9}{20}$$

$$\approx .24$$

$$F' = \frac{(s_2^2/\sigma_2^2)}{(s_1^2/\sigma_1^2)}$$

$$P(F' > f') = 5\%$$

$$= P\left(\frac{(s_2^2/\sigma_2^2)}{(s_1^2/\sigma_1^2)} > f'\right)$$

$$= P\left(\frac{s_2^2}{s_1^2}\boxed{\frac{\sigma_1^2}{\sigma_2^2}} > f'\right)$$

$$5\% = P\left(\frac{\sigma_1^2}{\sigma_2^2} > \frac{s_1^2}{s_2^2} f'\right)$$

$$95\% = P\left(\sigma_1^2/\sigma_2^2 < \underbrace{\frac{s_1^2}{s_2^2} f'}_{R_{big}}\right)$$

$$r_{big} = f' \cdot \frac{s_1^2}{s_2^2} = f' \cdot \frac{9}{20}$$

$$\approx 2 \cdot \frac{9}{20} = \frac{18}{20} = \frac{9}{10}.$$

90% confidence interval for σ_1^2/σ_2^2

is $\boxed{.24 < \sigma_1^2/\sigma_2^2 < .90}$