$X_1, X_2, \dots, X_n$ iid from distribution pdf $f(x;\theta)$

Life of a lightbulb follow exp. dist.

$f(x) = \lambda e^{-\lambda}$

$\hat{\Theta} = g(X_1, \dots, X_n)$ "estimator" for θ

what do we want from our estimators?

- Unbiased: $E[\hat{\Theta}] = \theta$
- Efficient: $Var(\hat{\Theta})$ small

} good.

- Consistent: as $n \to \infty$ $\hat{\Theta} \to \theta$
- Sufficient: no additional info from $X_1, \dots, X_n$ about θ not already encoded by $\hat{\Theta}$.

Def Given a sequence of random variables X_n, and a random variable X, we say that X_n converge in probability to X if $\forall \varepsilon > 0$

$$\lim_{n\to\infty} P(|X_n - X| < \varepsilon) = 1$$

Def $\hat{\Theta}$ is a consistent estimator for θ if $\hat{\Theta}$ converges in probability to θ.

Fact: if $\hat{\Theta}$ is unbiased and if $Var(\hat{\Theta}) \to 0$ as $n \to \infty$
then $\hat{\Theta}$ is a consistent estimator.

$$P(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

$$P(|\hat{\Theta}_n - \underset{\theta}{\underset{\shortparallel}{E[\hat{\Theta}_n]}}| < k\sqrt{Var\hat{\Theta}_n}) \geq 1 - \frac{1}{k^2}$$

$$s_n = \sqrt{Var\hat{\Theta}_n}$$

$$s_n \longrightarrow 0$$

$$P(|\hat{\Theta}_n - \theta| < \varepsilon) \geq 1 - \frac{1}{(\varepsilon^2/s_n^2)} = 1 - \frac{s_n^2}{\varepsilon^2}$$

$$k s_n = \varepsilon \qquad k = \frac{\varepsilon}{s_n}$$

$$\lim_{n\to\infty} P(|\hat{\Theta}_n - \theta| < \varepsilon) \geq \lim_{n\to\infty} 1 - \frac{s_n^2}{\varepsilon^2} = 1 \quad \square$$

Fact: If $\hat{\Theta}$ is asymptotically unbiased & $Var\,\hat{\Theta} \to 0$ as $n \to \infty$
$\Rightarrow \hat{\Theta}$ is consistent.

beginning: $P(|\hat{\Theta}_n - \theta| < \varepsilon)$

$$E[\hat{\Theta}_n] = \theta + b_n \quad \text{as } n \to \infty, \ b_n \to 0$$

$$|\hat{\Theta} - \theta| = |\hat{\Theta} - \theta - b_n + b_n| < \underbrace{|\hat{\Theta} - \theta - b_n|}_{\text{same argument}} + |b_n|$$

$$P(|\hat{\Theta}_n - \theta| < \varepsilon) \geq P(|\hat{\Theta} - \theta - b_n| + |b_n| < \varepsilon)$$

$$\geq P(|\hat{\Theta} - \theta - b_n| < \varepsilon/2,\ |b_n| < \frac{\varepsilon}{2})$$