

Plot:

Decide between hypotheses

H_0 "Null hypothesis" "Basic Assumption"
"innocent until proven guilty"

H_1 "Alternative hypothesis" "significant conclusion"
"beyond reasonable doubt"

Strategy: choose some sample statistic Y

and regions R_0 R_1

↓
accept H_0
reject H_1

↓
accept H_1
reject H_0

← "Test"

Suppose we want to test hypotheses about a population mean, assume normally distributed

$$H_0: \mu = 50$$

$$H_1: \mu = 85$$

$$n = 10$$

$$\sigma^2 = 120$$

$$\bar{X} = N\left(\mu, \frac{120}{10}\right)$$

look for a value x_{big} so that

if $\bar{X} \geq x_{\text{crit}}$ then reject H_0

$$P(\bar{X} \geq x_{\text{crit}} | H_0) \leq 0.001 = \alpha$$

$$H_0 \rightarrow \bar{X} = N(50, 12)$$

$$\frac{\bar{X} - 50}{\sqrt{12}} = z$$

$$\bar{X} = \sqrt{12}z + 50$$

$$P(z \geq z_{\text{crit}}) = 0.001$$

$$P(\bar{X} \geq \sqrt{12} z_{\text{crit}} + 50) = 0.001$$

$$R_1: \bar{x} \geq x_{\text{crit}}$$

$$R_0: \bar{x} < x_{\text{crit}}$$

$$P(\text{type 1 error}) = 0.001$$

$$P(\text{type 2 error})$$

$$P(\bar{X} < x_{\text{crit}} | H_1)$$

If we don't know variance

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \text{ is } t\text{-distributed random var. w/ } n-1 \text{ degrees of freedom.}$$

$$P(T \geq t_{\text{crit}} | H_0) = 0.001$$

$$H_0: \mu = 50$$

$$H_1: \mu = 85$$

statistic: T

$R_0: T < t_{\alpha/2}$

$R_1: T \geq t_{\alpha/2}$