$\mu_{y|x} = \alpha + \beta x$

$\boxed{\text{Part 4}}$ Normal Regression analysis

Additional assumption: for a given x, cond distribution

$f(y|x)$ normal w/ mean $\alpha + \beta x$

w/ variance σ^2 not depending on x

$$f(y|x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y - (\alpha+\beta x))^2}$$

$$\| $$

$$f(y|x; \alpha, \beta, \sigma^2)$$

get MLE statistics for α, β, σ^2

choose (for a given $\vec{x}, \vec{y}$ $\quad x_1, y_1, x_2, y_2 \ldots$)

want to maximize likelihood fcn.

$$L(\alpha, \beta, \sigma^2; \vec{x}, \vec{y}) = \prod f(y_i | x_i; \alpha, \beta, \sigma^2)$$

(in practice max $\ln L$)

$\bar{x} = \frac{1}{n} \sum x_i$

get estimates $\hat{\alpha}, \hat{\beta}, \hat{\sigma}^2$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} \qquad S_{xx} = \frac{1}{n} \sum (x_i - \bar{x})^2$$

$$S_{xy} = \frac{1}{n} \sum (x_i - \bar{x})(y_i - \bar{y})$$

$$\hat{\alpha} = \bar{y} - \frac{S_{xy}}{S_{xx}} \bar{x}$$

$$\hat{\sigma}^2 = \sum_{i=1}^{n} \left(y_i - (\hat{\alpha} + \hat{\beta} x_i)\right)^2$$

Regression analysis perspective

x_i's a fixed (not random vars)

but y_i's are

Ex Plants, examine ht after 1 day, 7 days, 14 days.

$x_1 = 1$ $\quad y_1 =$ ht at day 1

$x_2 = 7$ $\quad y_2$ 7

$x_3 = 14$ $\quad y_3 =$ - - - 14

from this perspective

sampling dist. for $\hat{\beta} = \frac{S_{xy}}{S_{xx}}$

$$= \frac{1}{S_{xx}} \frac{1}{n} \sum (x_i - \bar{x})(y_i - \bar{y})$$

α_i

normal

$\hat{\beta}$ is a normal random variable

$\mu_{\hat{\beta}} = \beta \qquad \sigma^2_{\hat{\beta}} = \frac{\sigma^2}{S_{xx}}$

$$\hat{\sigma}^2 = \sum_{i=1}^{n} \left(y_i - (\hat{\alpha} + \hat{\beta} x_i)\right)^2$$

<u>turns out:</u> $n\hat{\sigma}^2/\sigma^2$ is a χ^2_{n-2} dist. random var. and is indep. of $\hat{\beta}$.

consequently

$$\left(\frac{\hat{\beta}-\beta}{\sigma/\sqrt{S_{xx}}}\right) \Bigg/ \sqrt{\frac{\left(n\hat{\sigma}^2/\sigma^2\right)}{n-2}}$$

is a t dist. random var of $n-2$ degrees of freedom

$$= \left(\frac{\hat{\beta}-\beta}{\hat{\sigma}}\right) \underbrace{\sqrt{\frac{(n-2)S_{xx}}{n}}}_{\text{const.}}$$

Part 5 | Normal correlation analysis