$$P(a < X < b) = 95\%$$

$$X \longleftrightarrow \chi^2_n \qquad n = 20$$

$$\chi^2_{20} \approx \text{normal distribution w/}$$

$$\mu = n - 1 = 19$$

$$\sigma^2 = 2(n-1) = 38$$

If X is a χ^2_{20} random var then

$\sqrt{X}$ is a χ_{20} variable.

$$\mu(\sqrt{X}) \qquad \sigma^2(\sqrt{X})$$

$$\mu \qquad \sigma^2$$

$$P(c < \sqrt{X} < d) = 95\%$$

$$= P(c^2 < X < d^2) = 95\%$$

c, d?

$$\frac{\sqrt{X} - \mu(\sqrt{X})}{\sigma(\sqrt{X})} = Z$$

$$95\% = P(-2 < Z < 2) = P\left(-2 < \frac{\sqrt{\chi} - \mu(\sqrt{\chi})}{\sigma(\sqrt{\chi})} < 2\right)$$

$$= P\left(-2\sigma(\sqrt{\chi}) < \sqrt{\chi} - \mu(\sqrt{\chi}) < 2\sigma\sqrt{\chi}\right)$$

$$= P\left(-2\sigma(\sqrt{\chi}) + \mu(\sqrt{\chi}) < \sqrt{\chi} < 2\sigma(\sqrt{\chi}) + \mu(\sqrt{\chi})\right)$$

If $\sqrt{\chi}$ is a chi variable with n degrees of freedom

$$\mu(\sqrt{\chi}) = \sqrt{2}\,\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \qquad \sigma = \sqrt{n - \mu(\sqrt{\chi})^2}$$

Γ = gamma function

$n = 20$

$$\frac{\Gamma\left(\frac{21}{2}\right)}{\Gamma\left(\frac{20}{2}\right)} \approx 3.12$$

$$\mu(\sqrt{\chi}) = \sqrt{2}\,(3.12) \approx 4.42$$

$$\sigma(\sqrt{\chi}) = \sqrt{20 - (4.42)^2} \approx .68$$

$$95\% = P\left(-2\sigma(\sqrt{\chi}) + \mu(\sqrt{\chi}) < \sqrt{\chi} < 2\sigma(\sqrt{\chi}) + \mu(\sqrt{\chi})\right)$$

$$95\% = P(-2(.68) + 4.42 < \sqrt{x} < 2(.68) + 4.42)$$

$$= P(\underbrace{-1.36 + 4.42}_{3.06} < \sqrt{x} < 5.78)$$

$$P((3.06)^2 < X < (5.78)^2)$$

$$95\% = P(9.36 < X < 33.4)$$

$X \qquad \chi^2_{20} \qquad \mu(X) = 19$

$\sigma^2(X) = 38$

$$\frac{X - 19}{\sqrt{38}} \approx Z$$

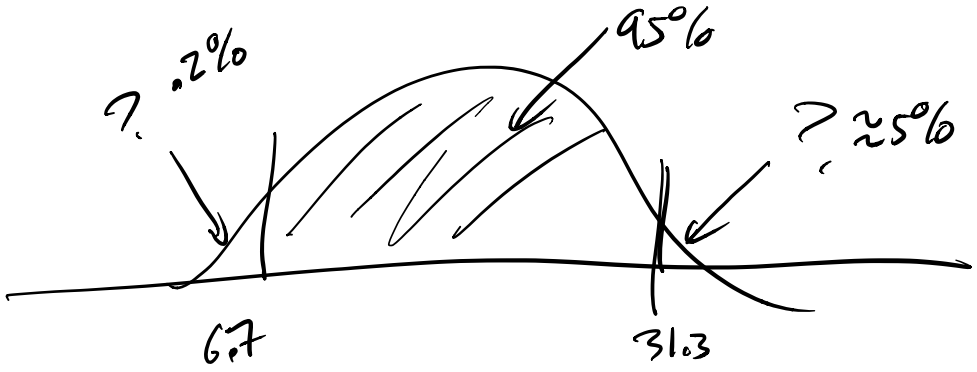
$$95\% = P(-2 < Z < 2)$$

$$= P\left(-2 < \frac{X - 19}{\sqrt{38}} < 2\right)$$

$$= P(19 - 2\sqrt{38} < X < 19 + 2\sqrt{38})$$

$$= P(19 - 12.3 < X < 19 + 12.3)$$

$$= P(6.7 < X < 31.3)$$



$\sqrt{2X}$ $\quad \mu = \sqrt{2n-1} \quad \sigma^2 = 1$

$95\% = P(-2 < \sqrt{2X} - \sqrt{2n-1} < 2)$

$= P(-2+\sqrt{39} < \sqrt{2X} < 2+\sqrt{39})$

$= P(4.24 < \sqrt{2X} < 8.24)$

$18 < 2X < 67.9$

$9 < X < 33.9$

$X \quad \chi^2_n \qquad \sqrt{2X}$

$n \gg 0$ this is close to normal

mean $\sqrt{2n-1}$

variance 1

$n = 20$

$95\% = P(-2 < Z < 2)$

$Z \approx \sqrt{2X} - \sqrt{2n-1}$

$= P(-2 < \sqrt{2X} - \sqrt{39} < 2)$

$= P(-2 < \sqrt{2X} - 6.2 < 2)$

$= P(4.2 < \sqrt{2X} < 8.2)$

$= P(17.6 < 2X < 67.2)$

$= P(8.8 < X < 33.6)$