If we have a normally dist. pop.

known σ^2, unknown μ.

$$Z = \frac{\bar{X} - \mu}{(\sigma/\sqrt{n})}$$ = standard normal random variable

$$P(-2 < Z < 2) = 95\%$$

$$\boxed{P\left(\bar{X} - \frac{2\sigma}{\sqrt{n}} < \mu < \bar{X} - \frac{2\sigma}{\sqrt{n}}\right) = 95\%}$$

$$\cancel{Z \overset{?}{\approx}} \frac{\bar{X} - \mu}{S/\sqrt{n}} \qquad S = \sqrt{S^2} = \frac{\sum_i (X_i - \bar{X})}{n-1}$$

not actually normal

turns out $\frac{\bar{X} - \mu}{S/\sqrt{n}}$ is a random var w/ the "t-distribution" with n-1 dgrees of freedom

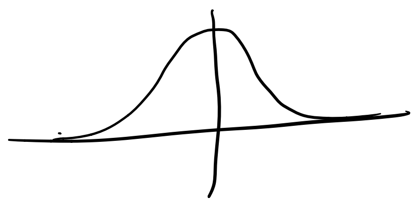
t-dist

$$f(t) = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{\pi k}\,\Gamma\left(\frac{k}{2}\right)} \cdot \left(1 + \frac{t^2}{k}\right)^{-\frac{k+1}{2}}$$

k = # of degrees of freedom

prop. to $$\frac{1}{(1+k^{-1}t^2)^{(k+1)/2}} \underset{\text{large } x}{\approx} \frac{1}{t^{k+1}}$$

normal $\simeq e^{-x^2} = \frac{1}{e^{x^2}}$ $\qquad \frac{1}{x^{k+1}}$



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normal population.

Claim pop mean is 20

Sample $n=6$ measure $\overline{X} = 21$ $\quad S = 1$

$$\frac{\overline{X} - \mu}{S/\sqrt{\cancel{6}\ 10}}$$

t-dist. ~~5~~ 9 degrees of freedom

$$\frac{21-20}{1/\sqrt{\cancel{6}\ 10}} = \sqrt{6} = 2.3 \text{ ish}$$

$\sqrt{10} \approx 3$

$P \approx$ between $2\frac{1}{2}$ & 5% that we observed

Table IV: Values of $t_{\alpha,\nu}$

ν	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$	ν
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10

.5% of measuring

$\bar{X} = 21$ & $S = 1$

if $\mu = 20$.